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Heterotic little string theories and holography

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Abstract: It has been conjectured that little string theories in six dimensions are holographic to critical string theory in a linear dilaton background. We test this conjecture for theories arising on the worldvolume of heterotic fivebranes. We compute the spectrum of chiral primaries in these theories and compare with results following from type-I-heterotic duality and the AdS/CFT correspondence. We also construct holographic duals for heterotic fivebranes near orbifold singularities. Finally we find several new little string theories which have $\mathrm{Spin}(32)/\mathbb{Z}_2$ or $E_8 \times E_8$ global symmetry but do not have a simple interpretation either in heterotic or M-theory.

Keywords: Superstrings and Heterotic Strings, p-branes, Conformal Field Models in String Theory.

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Contents

1. Introduction

One of the surprises of the second string revolution was the discovery of nontrivial Poincaré-invariant quantum theories in six dimensions [1]–[8]. Some of these theories are superconformal quantum field theories with $(2,0)$ or $(1,0)$ supersymmetry. Others, dubbed little string theories (LSTs), are not scale-invariant and do not have a stress-energy tensor or other local operators. They may have $(2,0)$, $(1,1)$, or $(1,0)$ supersymmetry. At length-scales much larger than the nonlocality scale they flow either to free super Yang-Mills theories, or to one of the nontrivial superconformal field theories mentioned above.

One way to construct LSTs is to consider flat parallel Neveu-Schwarz fivebranes in type-II or heterotic string theory and take the limit of zero string coupling. The theory on the branes remains interacting in this limit and defines an LST. Another way is to consider type-II or type-I string theory on an orbifold of the form $\mathbb{R}^4/\Gamma$ for some finite group $\Gamma \subset \mathrm{SU}(2)$. In the limit of zero string coupling the theory of twisted modes defines an LST. Finally, one may combine the two methods.

Using these methods the following classes of LSTs have been constructed:

(iia) $(1,1)$ LSTs obtained from IIA on ADE singularities [8];

(iib) $(2,0)$ LSTs obtained from IIB on ADE singularities [8];

(o) $(1,0)$ LSTs obtained from Spin(32)/$\mathbb{Z}_2$ heterotic fivebranes in flat space [8];

(e) $(1,0)$ LSTs obtained from $E_8 \times E_8$ heterotic fivebranes in flat space [8];

(ii′) $(1,0)$ LSTs obtained from IIA or IIB fivebranes at ADE singularities [9];

(o′) $(1,0)$ LSTs obtained from Spin(32)/$\mathbb{Z}_2$ heterotic fivebranes at ADE singularities [9];

(e′) $(1,0)$ LSTs obtained from $E_8 \times E_8$ heterotic fivebranes at ADE singularities [9].

These classes of LSTs are manifestly inequivalent as they have different supersymmetries and/or global symmetries. Some of these theories have alternative realizations; for example the LST obtained from IIA (IIB) string theory on an A_{N-1} singularity is equivalent to the LST of N IIB (IIA) fivebranes. Several other $(1,1)$ LSTs have been constructed in ref. [10]. They can be regarded as "excitations" of LSTs of type (iia) [8].

Above we defined LSTs by considering the decoupling limit of critical string theories. One can give a more intrinsic definition using Discrete Light Cone Quantization [6, 11, 12, 13]. Finding a Poincaré-invariant intrinsic definition of LSTs is an important open problem.

In ref. [14] it was conjectured that LSTs have a dual description in terms of critical string theory in linear dilaton backgrounds. This conjecture is similar in spirit to the AdS/CFT correspondence [15] which posits that certain conformally-invariant quantum field theories in d dimensions are dual to critical string theory or M-theory on backgrounds of the form $AdS_{d+1} \times X$ for some Einstein manifold X. Just as in the case of the AdS/CFT correspondence, stringy excitations are conjectured to be in one-to-one correspondence with operators in the boundary theory, and their S-matrix is the generating functional for the boundary correlators. The AdS/CFT correspondence becomes effective in the large-N limit, where string/M-theory reduces to supergravity. In contrast, the holographic conjecture of ref. [14] provides a framework in which the spectrum of operators in the boundary theory can be calculated for any N, but in general correlators cannot be computed. This is because the linear dilaton background by definition includes a strong coupling region where string perturbation theory fails. In some cases the strong-coupling singularity is resolved via M-theory, in which case one can compute the correlators for large N [14, 16].

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For theories of type (iia) and (iib) the relevant linear dilaton background is the familiar "throat" CFT describing the near-horizon physics of type-II fivebranes. The bosonic fields of this CFT consist of six free bosons describing coordinates parallel to the worldvolume of the branes, a boson describing the radial transverse coordinate, and an SU(2) Wess-Zumino-Witten (WZW) model describing the angular transverse coordinates. The holographic point of view reduces the classification problem for $(1,1)$ and $(2,0)$ LSTs to the classification of modular invariant partition functions for the WZW part of the CFT. It has been argued in refs. [14, 17] that LSTs obtained from type-II on ADE singularities correspond to ADE modular invariants of SU(2) WZW models [18]. Thus it appears that (iia) and (iib) theories represent a complete list of LSTs with sixteen supercharges. Furthermore, it has been checked in ref. [14] that the spectrum of worldsheet vertex operators in short representations of the space-time SUSY algebra includes all the expected moduli of the boundary LST. This supports the holographic conjecture.

In this paper we study heterotic $(1,0)$ LSTs using the holographic approach of ref. [14] (see also [19] for related work). In section 2 we construct the holographic description of heterotic fivebranes in flat space (LSTs of types (o) and (e)). We compute the spectrum of chiral primaries of the space-time supersymmetry algebra and compare with results known from the AdS/CFT correspondence and heterotic-type-I duality. Our analysis provides evidence that a single heterotic fivebrane has a nontrivial decoupling limit, unlike a single fivebrane in type-II. We find that all known operators of the heterotic LSTs (namely the moduli and their superpartners) have worldsheet counterparts.

LSTs of types (o) and (e) have global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ symmetry, where $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ or $E_8 \times E_8$, respectively. The $\mathrm{SU}(2)_R$ factor is the R-symmetry. In section 3 we discuss more general $(1,0)$ LSTs with this symmetry. For either choice of G the theories we construct are in one-to-one correspondence with affine Dynkin diagrams of types A,D, or E, with theories of types (o) and (e) being the A-series. As for type-II theories, the ADE scheme arises from the ADE classification of SU(2) WZW modular invariants. The D-series can be obtained by considering heterotic fivebranes at an A_1 singularity, i.e. they are a special class of theories of types (o′) and (e′). The exceptional E-series are a new type of LSTs which do not have a large N limit.[1] Since the spectrum of chiral operators in the D and E theories is rather complex, we only discuss a few interesting examples, leaving a full investigation for the future.

In general, $(1,0)$ heterotic LSTs need to have only $\mathrm{SU}(2)_R$ global symmetry. Examples of such theories can be obtained by orbifolding ADE heterotic LSTs by a finite group $\Gamma \subset \mathrm{SU}(2)_L \times G$. In section 4 we consider $\mathbb{Z}_n$ orbifolds of the A-

[1]We would like to emphasize that the ADE LSTs we are discussing here are not related to LSTs describing heterotic fivebranes near ADE orbifold singularities. The latter theories have smaller global symmetry groups.

series which describe heterotic fivebranes at A_{n-1} singularities. (The holographic description of type-II fivebranes at A_{n-1} singularities was discussed in ref. [20].) We derive the constraints following from the modular invariance of the worldsheet. Interestingly one can construct a consistent worldsheet description of an orbifold with an arbitrary embedding of the spin connection into the gauge connection. This means that the holographic description exists for an arbitrary LST of types (o′) and (e′). In contrast, for toroidal heterotic orbifolds (without fivebranes) modular invariance constrains the embedding in such a way as to ensure that the fivebrane charge of each orbifold plane vanishes. Section 5 contains our conclusions.

2. Holographic description of heterotic fivebranes in flat space

2.1 Supergravity solutions

The solution of the low-energy effective theory describing several coincident heterotic fivebranes (the "symmetric fivebrane") was constructed in ref. [21]. This solution is very similar to the one describing type-II fivebranes [22], but there are some slight differences in the interpretation of parameters, as argued below.

The symmetric fivebrane solution has the form

$$\begin{aligned} A_\mu &= -\rho_N^2 \bar{\Sigma}_{\mu\nu} \frac{2y^\nu}{y^2(y^2+\rho_N^2)}\,, \\ e^{2\phi} &= e^{2\phi_0} + \frac{N\alpha'}{y^2}\,, \\ H_{\mu\nu\lambda} &= -\epsilon_{\mu\nu\lambda}^{\ \ \ \ \rho}\nabla_\rho\phi\,, \\ ds^2 &= \eta_{ab}dx^a dx^b + e^{2\phi}dy^\mu dy^\mu\,. \end{aligned} \tag{2.1}$$

The notation here is as follows. The latin indices from the beginning of the alphabet run from 0 to 5 and are raised and lowered with Minkowski metric η_{ab}, the greek indices run from 0 to 3 and are raised and lowered with euclidean metric $\delta_{\mu\nu}$, y^2 means $y^\mu y^\mu$, $\rho_N^2 = N\alpha' \exp(-2\phi_0)$, and $\bar{\Sigma}_{\mu\nu}$ is the anti-self-dual t'Hooft tensor taking values in the Lie algebra of SU(2). It is understood that the SU(2) is minimally embedded into $\mathrm{Spin}(32)/\mathbb{Z}_2$ or $E_8\times E_8$, i.e. it is identified with one of the SU(2) groups in the $(\mathrm{Spin}(28)\times \mathrm{SU}(2)\times\mathrm{SU}(2))/\mathbb{Z}_2$ subgroup of $\mathrm{Spin}(32)/\mathbb{Z}_2$ or with the SU(2) group in the $E_8\times(E_7\times\mathrm{SU}(2))/\mathbb{Z}_2$ subgroup of $E_8\times E_8$.

The fivebrane charge of the solution eq. (2.1) (defined as the integral of H over an $\mathbf{S}^3$ $y^2 = R^2, R\to\infty$) is equal to N. Intuitively, eq. (2.1) describes a single SU(2) instanton of size ρ_N superimposed with $N-1$ small instantons (fivebranes) located at $y^\mu = 0$. This statement requires some qualifications. Since $dH = 0$, the integral of H over $y^2 = R^2$ is independent of R and equal to N. This means that if we define the number of fivebranes at $y = 0$ as the integral of H over a small $\mathbf{S}^3$ surrounding $y = 0$, we get N, and not $N-1$. However, this definition of the fivebrane charge is not unique. Note that H satisfies a Bianchi identity of the form

$$dH = \mathrm{Tr}\, R(\Omega_-) \wedge R(\Omega_-) - \mathrm{Tr}\, F \wedge F\,, \tag{2.2}$$

where Ω_- differs from the Levi-Civita connection ω by a torsion term, $\Omega_{-\mu}^{ab} = \omega_\mu^{ab} - H_\mu^{ab}$. One can define a new field strength H_0 which is a nonlinear function of H and ω and satisfies the more usual form of the Bianchi identity

$$dH_0 = \mathrm{Tr}\, R(\omega) \wedge R(\omega) - \mathrm{Tr}\, F \wedge F\,. \tag{2.3}$$

Using H_0 instead of H we get a different (gauge-invariant) definition of the fivebrane charge. For the symmetric fivebrane solution the new fivebrane charge measured at infinity is still N, while the fivebrane charge measured on a small $\mathbf{S}^3$ surrounding $y = 0$ is $N-1$. Given this nonuniqueness of the fivebrane charge, we prefer to fix the precise relation between N and the number of fivebrane at the origin N_5 by matching symmetries and operators of the worldsheet CFT and the boundary LST. We will see below that this leads to the identification $N = N_5 + 1$.[2]

We are interested in the decoupling limit $\phi_0 \to -\infty$ when the asymptotic string coupling goes to zero. If we introduce the radial coordinate $t = \log \sqrt{y^2}$ and rescale $x^a \to x^a (N\alpha')^{1/2}$, the limiting solution takes the form

$$\begin{aligned} A_\mu &= -\bar{\Sigma}_{\mu\nu} \frac{2y^\nu}{y^2}, \\ \phi &= -t, \\ H &= -N\epsilon, \\ ds^2 &= N\alpha' \left(\eta_{ab} dx^a dx^b + dt^2 + ds_3^2 \right), \end{aligned} \tag{2.4}$$

where ds_3^2 is the standard round metric on the hypersurface $y^2 = 1$ and ϵ is its volume form. Note that the transverse coordinates y^μ now parametrize a copy of $\mathbf{S}^3 \times \mathbb{R}$ with N units of H-flux on $\mathbf{S}^3$. The size of the single instanton goes to infinity in the decoupling limit, and the gauge field A_μ becomes a flat connection on $\mathbf{S}^3 \times \mathbb{R}$. Since $\mathbf{S}^3 \times \mathbb{R}$ is simply connected, A_μ can be gauged away. A priori, one could expect this supergravity background to solve the stringy equations of motion only if the dilaton gradient is small ($\ll \alpha'^{-1/2}$), which requires $N \gg 1$. However, it is well known that eq. (2.4) corresponds to a soluble worldsheet CFT [21] and therefore is an exact solution for all N.

It has been proposed in ref. [14] that string theory in a background of the form

$$\begin{aligned} ds^2 &= \alpha' \left(\eta_{ab} dx^a dx^b + dt^2 + g_{ij} dy^i dy^j \right), \\ \phi &= -\frac{Qt}{2} \end{aligned} \tag{2.5}$$

[2]Note that in the type-II case the relation between N and N_5 is simply $N = N_5$.

where η_{ab} is the Minkowski metric on $\mathbb{R}^{d-1,1}$, is equivalent to a quantum theory without gravity on $\mathbb{R}^{d-1,1}$. This equivalence is holographic in the sense that the latter theory lives at $t = -\infty$ and its correlators can be computed from the S-matrix of string excitations in the background eq. (2.5). In particular, global symmetries of the "boundary" theory correspond to gauge symmetries of the "bulk" theory. Of course, in order to compute the S-matrix string perturbation theory is not sufficient, as the excitations propagating towards $t = -\infty$ inevitably reach the region of strong coupling. But in order to compute the *spectrum* of excitations it is sufficient to analyze the theory near the perturbative region $t = +\infty$. (This analysis can miss some purely nonperturbative states [14]. We will see examples of this below.)

Following the proposal of ref. [14], we conjecture that the heterotic string background eq. (2.4) is holographic to the worldvolume theory of heterotic fivebranes located at $t = -\infty$. The parameter N is related to the number of fivebranes N_5 living at $t = -\infty$ via $N = N_5 + 1$.

2.2 Worldsheet partition function

The worldsheet CFT corresponding to the heterotic background eq. (2.4) has been worked out in ref. [21]. The bosonic fields are six free bosons $X^a, a = 0, \ldots, 5$, an SU(2) WZW model at level $k = N$, and a Feigin-Fuchs field t with real background charge $\sqrt{2/N}$. The level of the WZW model is simply the flux of the H-field through $\mathbf{S}^3$. The fermionic fields are ten right-moving fermions and 32 left-moving fermions. Not all fermions are free: four of the left-movers and four of the right-movers are coupled to certain (unequal) pure gauge connections. Performing the gauge rotation, it possible to make the fermions free. This rotation is anomalous and shifts the level of the WZW from $k = N$ to $k = N - 2$. Thus we end up with ten free right-moving fermions and 32 free left-moving fermions. The difference between $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ and $G = E_8 \times E_8$ is encoded in the boundary conditions for the left-moving fermions. It is convenient to bosonize the left-movers into sixteen free bosons H_α, $\alpha = 1, \ldots, 16$ living on the weight lattice of G.

The $(4, 0)$ worldsheet superalgebra combines six of the right-moving fermions ψ^a with X^a, and the remaining right-moving fermions ψ^μ with t and the WZW field $g(z, \bar{z})$. The gauged worldsheet supercurrent is right-moving and has the form

$$G(z) = \eta_{ab}\psi^a \partial X^b + \psi^0 \partial\phi + \sqrt{\frac{2}{N}}\left[J^i_{WZW}\psi^i + \psi^1\psi^2\psi^3 - \partial\psi^0\right]. \tag{2.6}$$

Here and in what follows the latin indices from the middle of the alphabet $i, j, \ldots$ run from 1 to 3. The partition function of this CFT on a torus is

$$\begin{aligned} Z(\tau,\bar{\tau}) = {} & \frac{1}{2}(\mathrm{Im}\ \tau)^{-2}\eta(\bar{\tau})^{-20}\ \eta(\tau)^{-8}\Big(\theta_3(q)^4 - \theta_4(q)^4 - \theta_2(q)^4\Big) \times \\ & \times Z_\phi(\tau,\bar{\tau}) Z_{WZW}(\tau,\bar{\tau})\Theta_G(\bar{\tau}) \,. \end{aligned} \tag{2.7}$$

Here Z_ϕ and Z_{WZW} are the partition functions of the Feigin-Fuchs field and the SU(2) WZW model, respectively, and Θ_G is the theta-function of the (even, self-dual) weight lattice of G. It is easy to check that $Z(\tau,\bar{\tau})$ is modular-invariant provided $Z_{WZW}(\tau,\bar{\tau})$ is a modular-invariant combination of affine SU(2) characters.

A celebrated theorem [18] states that such modular-invariant combinations have an ADE classification. Only the A-type modular invariant exists for all levels k, and thus it is very plausible that it corresponds to heterotic fivebranes in flat space. We will discuss the interpretation of D and E invariants in the next section.

2.3 Symmetries of the worldsheet CFT

Let us now discuss the symmetries of the worldsheet CFT and the corresponding LSTs. The internal CFT has $(4,0)$ worldsheet supersymmetry. This implies [23] that the space-time theory has $(1,0)$ supersymmetry in six dimensions. The corresponding SU(2) R-current is right-moving and has the form

$$J^i_R(z) = J^i_{WZW}(z) + \frac{1}{2}\varepsilon^{ijk}\psi^j\psi^k\,, \qquad i = 1,2,3\,. \tag{2.8}$$

Its level is $k+2$.[3] The supercharges transform in the $(\mathbf{4},\mathbf{2})$ of $\mathrm{Spin}(1,5)\times \mathrm{SU}(2)_R$, where $\mathrm{Spin}(1,5)$ is the six-dimensional Lorenz group. There is also a left-moving $\mathrm{SU}(2)_L$ current $J_{WZW}(\bar{z})$ with level k and a left-moving G current $J_G(\bar{z})$ with level 1 obtained from the H_α using the Frenkel-Kac-Segal construction. The supercharges are inert under $\mathrm{SU}(2)_L\times G$.

The construction of the supercharges is standard [23]. Bosonizing right-moving fermions ψ^a and ψ^μ we can construct spin operators S_α and Σ_A, where α is the index of the $\mathbf{4}$ representation of $\mathrm{Spin}(1,5)$ and A is the index of the $\mathbf{2}$ representation of $\mathrm{SU}(2)_R$. Let σ be the bosonized superconformal ghost. In the $-1/2$ picture the supercurrent is given by

$$Q_{\alpha A}(z) = e^{-\sigma(z)/2}S_\alpha(z)\Sigma_A(z)\,. \tag{2.9}$$

The GSO projection is imposed by requiring that physical vertex operators be local with respect to $Q_{\alpha A}(z)$.

By the usual "holographic" logic, the boundary LST must have $(1,0)$ SUSY and $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times G$ global symmetry. The $\mathrm{SU}(2)_R$ symmetry is the R-symmetry of the $(1,0)$ theory. This is the expected result for a theory of heterotic fivebranes in flat space. The symmetry $\mathrm{SO}(4)\simeq\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ corresponds to the rotations of the directions transverse to the fivebranes.

The case $k=0$ is somewhat special in that the WZW part is absent. Thus the $\mathrm{SU}(2)_L$ symmetry acts trivially in this case. This fact together with type-I-heterotic duality provides a check on our identification $k = N-2 = N_5-1$. For

[3] The right-moving current $J_{WZW}(z)$ is not BRST-invariant.

$G = \mathrm{Spin}(32)/\mathbb{Z}_2$ we can think of N_5 heterotic fivebranes as N_5 type-I D5 branes. (The type-I description is not suitable for discussing the decoupling limit, but it is good enough to identify symmetries and flat directions.) The worldvolume theory of N_5 D5 fivebrane reduces at low energies to super Yang-Mills theory with gauge group $\mathrm{Sp}(N_5)$, 32 hypermultiplets in the fundamental representation, and a hypermultiplet in the antisymmetric tensor representation. The antisymmetric tensor is a doublet of $\mathrm{SU}(2)_L$, while the rest of the fields are singlets. For $N_5 = 1$ the antisymmetric tensor is decoupled from the rest of the theory (it describes the center-of-mass motion of the D5 brane), and $\mathrm{SU}(2)_L$ acts trivially on the low-energy degrees of freedom. According to our identification, precisely at this value of N_5, the $\mathrm{SU}(2)_L$ action on the worldsheet becomes trivial, so apparently $\mathrm{SU}(2)_L$ acts trivially on the full LST, and not just on the low-energy degrees of freedom.

2.4 The spectrum of space-time chiral primaries

The holographic worldsheet description of heterotic LSTs found above can be used to compute the spectrum of operators in chiral multiplets of the space-time SUSY algebra. For $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ we will compare our results to the spectrum of chiral operators in the super Yang-Mills theory living on type-I D5 branes. For $G = E_8 \times E_8$ we will compare to the results obtained using the AdS/CFT correspondence [15, 24, 25].

The space-time chiral operators look very much like in the type-II LSTs [14]. For simplicity, we will limit ourselves to space-time scalars. Consider the following ansatz for the "internal" part of the Neveu-Schwarz vertex operators (in the -1 picture):

$$V_{j_L,j_R}(z,\bar{z}) \sim e^{-\sigma(z)} W_{j_L}(\bar{z}) \psi^i(z) V_{j_R}(z) e^{\beta\phi(z,\bar{z})} \,. \tag{2.10}$$

Here $V_{j_R}(z)$ is a primary of the right-moving bosonic SU(2) current algebra with SU(2) spin j_R, and $W_{j_L}(\bar{z})$ is a primary of the left-moving Virasoro algebra with $\mathrm{SU}(2)_L$ spin j_L. We need to require that this operator be a primary of the gauged supercurrent $G(z)$. One can show that this condition is met if the $\mathrm{SU}(2)_R$ indices of the fermion ψ^i and of V_{j_R} are contracted to form the representation $\mathbf{2j_R + 3}$ or $\mathbf{2j_R - 1}$. In order for V_{j_L,j_R} to be a vertex operator for a physical state, it must have dimension $(1,1)$. This is achieved by setting $\beta = j_R\sqrt{2/N}$ and taking $W_{j_L}(\bar{z})$ to be a Virasoro primary of dimension

$$h_L = 1 + \frac{j_R(j_R+1)}{N} \,. \tag{2.11}$$

It is easy to check that V_{j_L,j_R} is local with respect to $Q_{\alpha A}$. Moreover, one can check that it is space-time chiral, i.e. acting on it with a space-time supercharge does not produce operators with higher $\mathrm{SU}(2)_R$ spin. Moreover, out of the two "physical" contractions of $\mathrm{SU}(2)_R$ indices of ψ^i and V_{j_R} only the one with $\mathrm{SU}(2)_R$ spin $j_R + 1$ is a primary (the other one is its descendant). Consequently, for any j_L and any acceptable $W_{j_L}(\bar{z})$ we get precisely one chiral primary of the space-time SUSY algebra.

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The A_{N-1} modular invariant at level $k = N-2$ contains affine $\mathrm{SU}(2)_k$ primaries of the form $(\mathbf{2j+1}, \mathbf{2j+1})$ with $2j = 0, 1, \ldots, N-2$. Then eq. (2.11) suggests a natural way to get a suitable W_{j_L}: one has to take a left-moving primary $V_{j_R}(\bar{z})$ and get its level-one descendant with respect to the left-moving current algebra. The full left-moving current algebra is $\mathrm{SU}(2)_L \times \mathbb{R} \times G$, where the $\mathbb{R}$-current is $\partial\phi$.

If W_{j_L} is the descendant with respect to the G current algebra, then $j_L = j_R$ and W_{j_L} is automatically a Virasoro primary. In this case the operator eq. (2.10) transforms as $(\mathbf{2j+1}, \mathbf{2j+3})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and is in the adjoint of G.

If W is a descendant with respect to the $\mathrm{SU}(2)_L \times \mathbb{R}$ current algebra, then depending on how $\mathrm{SU}(2)_L$ indices are contracted we get one operator with $j_L = j_R + 1$, one with $j_L = j_R - 1$, and two with $j_L = j_R$. The first two are automatically physical (i.e. the left-moving part is a Virasoro primary). One linear combination of the other two is also physical. Thus we get three physical operators, $(\mathbf{2j_R+3}, \mathbf{2j_R+3})$, $(\mathbf{2j_R+1}, \mathbf{2j_R+3})$, and $(\mathbf{2j_R-1}, \mathbf{2j_R+3})$. All are singlets with respect to G.

To summarize, the theory has G-singlet chiral primaries of the form $(\mathbf{2j+3-2n}, \mathbf{2j+3})$ with $2j = 0, 1, \ldots, N-2$, $n = 0, 1, 2$, and chiral primaries in the adjoint of G of the form $(\mathbf{2j+1}, \mathbf{2j+3})$ with $2j = 0, \ldots, N-2$. Note that $(\mathbf{2j+3}, \mathbf{2j+3})$ is a traceless symmetric tensor of $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ with $2j+2$ indices. These operators are in one-to-one correspondence with Casimirs of $\mathrm{SU}(N)$.

Comparison with type-I D5 branes Let us now compare these operators to the chiral operators of the super Yang-Mills theory on $k+1 = N-1 = N_5$ type-I D5 branes (for $G = \mathrm{Spin}(32)/\mathbb{Z}_2$). The fivebrane gauge group is $\mathrm{Sp}(k+1)$, and the hypermultiplet content is given by

$$q \sim (\mathbf{1}, \mathbf{2}, \mathbf{32}, \mathbf{2k+2}), \qquad Y \sim (\mathbf{2}, \mathbf{2}, \mathbf{1}, (\mathbf{k+1})(\mathbf{2k+1}) - \mathbf{1}), \\ \text{of } \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{Spin}(32)/\mathbb{Z}_2 \times \mathrm{Sp}(k+1), \tag{2.12}$$

where Y is the antisymmetric "traceless" tensor of $\mathrm{Sp}(k+1)$.

Let us begin with the case $k = 0$. In this case the only gauge-invariant chiral primary operator is qq where the $\mathrm{SU}(2)_R$ indices are symmetrized and $\mathrm{Spin}(32)/\mathbb{Z}_2$ indices are antisymmetrized. This operator transforms as $\mathbf{3}$ of $\mathrm{SU}(2)_R$ and as the adjoint of $\mathrm{Spin}(32)/\mathbb{Z}_2$.[4] This agrees precisely with what we found from the worldsheet. To see this note that for $k = 0$ the left-moving $\mathrm{SU}(2)_L$ current is absent, and most of the worldsheet operators listed above are absent too.

For $k > 0$ the Yang-Mills theory has chiral primary operators in the adjoint of $\mathrm{Spin}(32)/\mathbb{Z}_2$ of the form $qY^pq, p = 0, \ldots, k$. They transform as $(\mathbf{p+1}, \mathbf{p+3})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and match precisely all the operators in the adjoint of $\mathrm{Spin}(32)/\mathbb{Z}_2$ that we found on the worldsheet. The Yang-Mills spectrum also includes singlets of

[4]Incidentally, this operator is the lowest component of a supermultiplet containing the $\mathrm{Spin}(32)/\mathbb{Z}_2$ global current.

$\mathrm{Spin}(32)/\mathbb{Z}_2$ of the form $\mathrm{Tr}\, Y^{p+2}, p = 0, \ldots, k-1$ which transform as $(\mathbf{p+3}, \mathbf{p+3})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. They also have worldsheet counterparts. However, there are other operators in the LST which do not have counterparts in Yang-Mills theory. Firstly, there are singlets of $\mathrm{Spin}(32)/\mathbb{Z}_2$ with unequal $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ spins. Secondly, there is a $\mathrm{Spin}(32)/\mathbb{Z}_2$-singlet which transforms as $(\mathbf{k+3}, \mathbf{k+3})$ of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

The latter operator is interesting. It has the same worldsheet origin as the operators which match $\mathrm{Tr}\, Y^{p+2}, p = 0, \ldots, k-1$ and its quantum numbers are the same as those of $\mathrm{Tr}\, Y^{p+2}$ for $p = k$. Why is this operator absent in the super Yang-Mills spectrum? We see three possible explanations for this.

1. In general the operators of the low-energy Yang-Mills theory are a subset of the operators of the LST. Thus it is not surprising that there is no one-to-one match between them. In fact, examples of this sort already appear for LSTs with sixteen supercharges studied in ref. [14]. If this interpretation is correct, then $(\mathbf{k+3}, \mathbf{k+3})$ does not correspond to a flat direction of the LST, unlike $(\mathbf{p+3}, \mathbf{p+3}), p = 0, \ldots, k-1$. Unfortunately we do not know how to check this using worldsheet methods.

2. Recall that strictly speaking the symmetric fivebrane solution describes $k+2$ fivebranes with one fivebrane blown up to a finite size ρ_N. ρ_N goes to infinity in the decoupling limit, which means that we are infinitely far along the Higgs branch in the direction where the gauge group $\mathrm{Sp}(k+2)$ is broken down to $\mathrm{Sp}(k+1)$. The effective low-energy Yang-Mills theory includes all the fields of the $\mathrm{Sp}(k+1)$ system plus a decoupled singlet which describes the relative position of the $k+1$ small fivebranes and the remaining large fivebrane. We could incorporate this by working with a field $\tilde{Y}$ which is an antisymmetric tensor of $\mathrm{Sp}(k+2)$, in which case the independent operators are $\mathrm{Tr}\, \tilde{Y}^{p+2}, p = 0, \ldots, k$. Naively, we would expect these operators to be expressible as polynomials in $\mathrm{Tr}\, Y^p$ and the decoupled singlet. The fact that this factorization does not occur on the worldsheet can be interpreted as the failure of decoupling in the full LST. If this is correct, then the LST we are studying is *not* the same as the LST of $k+1$ fivebranes at the origin of their moduli space.

3. The holographic conjecture is wrong.

In our view, the third possibility is unlikely. To decide between the first two possibilities one has to establish whether there is a flat directions along which $(\mathbf{k+3}, \mathbf{k+3})$ has a vacuum expectation value.

Comparison with AdS/CFT If we take $G = E_8 \times E_8$, then the corresponding LST is believed to flow in the infrared to an interacting $(1,0)$ superconformal field theory. Its large N limit is described by supergravity on $AdS_7 \times \mathbf{S}^4/\mathbb{Z}_2$ [24]. To describe the $\mathbb{Z}_2$ action it is best to think of $\mathbf{S}^4$ as a hypersurface $x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 1$

in $\mathbb{R}^5$; then $\mathbb{Z}_2$ acts by $x_5 \to -x_5$ and by flipping the sign of the M-theory 3-form. The fixed set of the $\mathbb{Z}_2$ action is an $\mathbf{S}^3 \subset \mathbf{S}^4$. Kaluza-Klein excitations on AdS_7 are believed to be in one-to-one correspondence with operators of the large N $(1,0)$ field theory on the boundary of AdS_7. The spectrum of Kaluza-Klein excitations has been computed in [25]. To make a meaningful comparison, it is useful to keep in mind that although every chiral primary of the superconformal algebra at large N is expected to correspond to a chiral operator in the LST, the converse is not necessarily true. Some chiral operators in the LST may acquire large (i.e. divergent in the large N limit) dimensions as one flows to the infrared and will not be present in the Kaluza-Klein spectrum.

One can easily see that all chiral primaries charged under $E_8 \times E_8$ found in ref. [25] are reproduced by our worldsheet computation. The traceless symmetric tensors of $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ with orders $2, 3, \ldots$ also appear in the large N analysis. Of course, the truncation of the spectrum at order N cannot be seen at the level of supergravity. However, most chiral primaries found in ref. [25] (those which contain odd place-holder fields; see ref. [25] for details) do not seem to have a "worldsheet" counterpart. After a little thought, this may not be so surprising. The existence of operators with odd place-holders is due to the presence of the hidden eleventh dimension in the $E_8 \times E_8$ heterotic string at strong coupling. One cannot expect to see this hidden dimension by doing a worldsheet CFT computation. In particular, all operators which are self-dual 3-forms are missed in the worldsheet approach. Similar phenomena are observed in $(2,0)$ LSTs [14]. In the type-II case, however, it may be possible to find the missing states by considering D0-branes of the throat CFT [14]. This has no analogue in the heterotic case, since the heterotic "D0-charge" (the momentum along the eleventh dimensions) is not conserved.

Similarly, for $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ we do not see any evidence for $\mathrm{Sp}(k+1)$ gauge symmetry in our worldsheet approach, because the gauge group arises through non-perturbative effects (see also [19]). The spectrum of chiral operators is nevertheless consistent with heterotic-type-I duality, as discussed above.

3. More general heterotic LSTs with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ symmetry

3.1 Worldsheet partition functions

In the previous section we discussed the worldsheet description of heterotic fivebranes for which the partition function of the WZW model was of the A-type. However, we can replace the WZW partition function in eq. (2.7) by any other modular invariant WZW partition function. Such partition functions have an ADE classification [18], so we can replace the A-type partition function in eq. (2.7) by a partition function of D- or E-type. The total partition function will then be modular invariant and we obtain additional models with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ global symmetry and $(4,0)$ worldsheet supersymmetry. The A-type partition functions exist for all levels k, the D-type modular invariants require even k, and the E-type partition functions, E_6, E_7, E_8, exist for $k = 10, 16, 28$.

The models with D-type WZW partition functions have a simple space-time interpretation. Recall that the D-type modular invariants can be regarded as Z_2 orbifolds of the A-type modular invariants at the same level. The Z_2 in question acts on left-moving SU(2) primaries as the center of SU(2). In particular it acts on the group-valued field $g(z, \bar{z})$ of the WZW model as

$$g(z, \bar{z}) \to -g(z, \bar{z})\,. \tag{3.1}$$

Consequently this orbifolding corresponds to reflecting all four coordinates transverse to the fivebranes. Thus the D-series describe fivebranes at an $\mathbb{R}^4/\mathbb{Z}_2$ singularity, i.e. they correspond to a particular class of theories of types (o′) and (e′). Note that the $\mathbb{Z}_2$ orbifold action here does not involve the internal left-moving bosons, which implies that the gauge connection has a trivial monodromy.[5] By virtue of the anomalous Bianchi identity $dH = \mathrm{Tr}\, R \wedge R - \mathrm{Tr}\, F \wedge F$ such an orbifold carries nonzero fivebrane charge. The usual perturbative $\mathbb{R}^4/\mathbb{Z}_2$ orbifold, on the other hand, has the spin connection embedded in the gauge connection in a particular manner so that the fivebrane charge cancels between $\mathrm{Tr}\, R \wedge R$ and $\mathrm{Tr}\, F \wedge F$ and the group G is broken to a subgroup. We will discuss fivebranes near this and other orbifolds in the next section.

We still need to determine the precise relation between the number of fivebranes N_5 and the WZW level k. Analogy with the previous section suggests that k is the total fivebrane charge counted on the cover up to a shift, i.e. $k = 2(N_5 - a)$ for some constant a. For $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ we can try to determine a by examining the type-I dual of our brane configuration. It consists of N_5 type-I D5 branes near an $\mathbb{R}^4/\mathbb{Z}_2$ singularity, with $\mathbb{Z}_2$ acting trivially on the gauge bundle. Unlike in the case of type-I fivebranes in flat space, at the origin of the moduli space the fivebrane theory flows to an interacting $(1,0)$ fixed point [27, 28]. This theory has a Higgs branch and, for $N_5 \geq 4$, also a Coulomb branch. The physics of the Higgs branch is "exotic", i.e. it cannot be described by a weakly coupled field theory. The physics of the Coulomb branch is described by super Yang-Mills theory coupled to a tensor multiplet. The Yang-Mills theory has gauge group $\mathrm{Sp}(N_5) \times \mathrm{Sp}(N_5 - 4)$, 16 hypermultiplets q in $(\mathbf{2N_5}, \mathbf{1})$, and one hypermultiplet Q in $(\mathbf{2N_5}, \mathbf{2N_5 - 8})$ [27, 28]. The global symmetry of this theory is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{Spin}(32)/\mathbb{Z}_2$, where the lowest components of

[5] The D-series heterotic CFT we are discussing can be obtained from the type-II CFT describing fivebranes near an $\mathbb{R}^4/\mathbb{Z}_2(-1)^{F_L}$ orbifold [14, 20] by embedding the spin connection into the gauge connection [26]. When applied to the type-II CFT describing fivebranes near an $\mathbb{R}^4/\mathbb{Z}_2$ orbifold, this procedure yields a slightly different heterotic CFT discussed at the end of section 4. Its symmetry group coincides with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ locally, but not globally.

q transform as $(\mathbf{1}, \mathbf{2}, \mathbf{32})$ and the lowest components of Q transform as $(\mathbf{2}, \mathbf{2}, \mathbf{1})$. For $N_5 = 4$ the Yang-Mills theory reduces to $\mathrm{Sp}(N_5)$ with 16 hypermultiplets in the fundamental representation, and therefore the $\mathrm{SU}(2)_L$ symmetry acts trivially on the Coulomb branch. In the worldsheet theory $\mathrm{SU}(2)_L$ disappears for $k = 0$, while for $k > 0$ the $\mathrm{SU}(2)_L$ symmetry clearly acts nontrivially. This suggests that $k = 2(N_5 - 4)$. Although it is plausible that for $G = E_8 \times E_8$ the answer is the same, we do not know how to show this.

The theories with E-type WZW partition functions do not have a simple geometric origin. It is easy to see that they cannot be obtained by orbifolding the heterotic fivebrane theory: any orbifold except the $\mathbb{R}^4/\mathbb{Z}_2$ that gives rise to the D-series would break the $\mathrm{SU}(2)_L \times G$ symmetry. On the other hand, the E-series LSTs have the full $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$ symmetry group. This implies that the E-series, unlike the D-series, cannot be obtained as a geometric orbifold of the A-series. Thus the E-series are new heterotic LSTs. Since they arise at $k = 10, 16, 28$, and the level is related to the flux of the H-field through the 3-sphere surrounding the fivebranes via $k = N - 2$, these theories describe systems of $12, 18$, and 30 fivebranes, respectively. However, it is not possible to think of these fivebrane configurations in terms of low-energy supergravity, since the dilaton gradient (=background charge of the Feigin-Fuchs field) is of order one in string units.

We will not perform an exhaustive analysis of the chiral primary operators of the D and E models. Instead we discuss a few examples in the next subsection. For large k it should also be possible to study the D-series LSTs with $G = E_8 \times E_8$ using the AdS/CFT correspondence, but this has not been done yet. The E-series theories exist only for small values of k, so the worldsheet approach is the only possibility in this case.

3.2 Examples of chiral primaries

The D-series The modular invariant of type $D_{N/2+1}$ with N even contains "untwisted" primaries of the form $(\mathbf{2j+1}, \mathbf{2j+1})$ with $2j = 0, 2, 4, \ldots, N-2$ and "twisted" primaries of the form $(\mathbf{N/2+2n}, \mathbf{N/2-2n})$ with integer n. The left-right symmetric primaries are the easiest to analyze, as they behave in the same way as the primaries of the A-series. The "untwisted" primaries are all left-right symmetric and produce the same spectrum of chiral operators as the A-series, except that $2j$ is restricted to be even. In particular, we obtain chiral primaries which are traceless symmetric tensors of $\mathrm{SO}(4)$ of even orders $2, 4, \ldots, N$. They correspond to the Casimirs of $\mathrm{SO}(N+2)$ of the form $\mathrm{Tr}\, X^{2k}, k = 1, \ldots, N/2$. Among the "twisted" primaries there is a single left-right symmetric one, $(\mathbf{N/2}, \mathbf{N/2})$. It yields a chiral primary operator which is a traceless symmetric tensor of $\mathrm{SO}(4)$ of order $N/2+1$. Its order matches the order of the pfaffian of $\mathrm{SO}(N+2)$. It also yields a chiral primary $(\mathbf{N/2}, \mathbf{N/2+2})$ which transforms in the adjoint of G.

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We will not describe the complete spectrum of chiral operators arising from the left-right asymmetric WZW primaries. Instead, let us consider a simple example, that of $(\mathbf{N/2+2, N/2-2})$. In this case the condition eq. (2.11) is satisfied only if we take $W_{j_L}(\bar{z})$ to be the primary of the left-moving bosonic SU(2) with spin $(N+2)/4$. Then we get a space-time chiral primary with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ content $(\mathbf{N/2+2, N/2})$.

For $G = E_8 \times E_8$ the theory of N_5 heterotic fivebranes at an $\mathbb{R}^4/\mathbb{Z}_2$ singularity flows to an interacting $(1,0)$ fixed point [9]. At large N_5 this fixed point is expected to be holographic to supergravity on $AdS_4 \times \mathbf{S}^4/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, where the two $\mathbb{Z}_2$'s act by $x_5 \to -x_5$ and $x_i \to -x_i, i = 1, \ldots, 4$. It would be interesting to study the Kaluza-Klein spectrum in this case and compare it to the one found above. An analogy with the type-II case [14] suggests that chiral operators coming from the "pfaffian" worldsheet primary $(\mathbf{N/2, N/2})$ have a nonperturbative origin: they are related to an M2-brane wrapped around submanifolds of $\mathbf{S}^4/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. The fact that one of these operators is in the adjoint of $E_8 \times E_8$ suggests that the corresponding submanifold has boundaries [29].

The E-series The E-series modular invariants have left-right symmetric primaries with the following spins:

$$\begin{aligned} E_6: &\quad 2j = 0, 3, 4, 6, 7, 10\,, \\ E_7: &\quad 2j = 0, 4, 6, 8, 10, 12, 16\,, \\ E_8: &\quad 2j = 0, 6, 10, 12, 16, 18, 22, 28\,. \end{aligned} \tag{3.2}$$

A primary with spin j yields, among others, a chiral operator which is a singlet of G and transforms with respect to SO(4) as a traceless symmetric tensor of order $2j+2$. It is easy to see that the orders of these tensors match the orders of the Casimirs of the respective Lie algebras.

4. Fivebranes near orbifold singularities

In this section we construct heterotic LSTs whose global symmetry is a proper subgroup of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$. The simplest way to obtain such theories is to orbifold the ADE LSTs discussed above by a finite group $\Gamma \subset \mathrm{SU}(2)_L \times G$ (if one wants to preserve $(1,0)$ SUSY, $\mathrm{SU}(2)_R$ has to be a spectator.) Since $\mathrm{SU}(2)_L$ acts on the fundamental WZW field $g(z, \bar{z})$ by left multiplication, one can interpret the orbifolded LSTs as representing fivebranes near an $\mathbb{R}^4/\Gamma$ orbifold. However, this interpretation can be taken literally only when the dilaton gradient in the theory is small, i.e. for large N. In particular, orbifolds of E_n LSTs cannot be understood in geometric terms. Furthermore, for special values of the WZW level the worldsheet CFT has accidental symmetries, in which case Γ can be chosen to be a subgroup of this larger symmetry group.[6] For example, $\mathrm{SU}(2)_4$ WZW corresponding to the modular invariant of type D_4 has accidental $\mathrm{SU}(3)_1$ symmetry [30]. In this paper we limit ourselves to studying $\mathbb{Z}_n$ orbifolds of the A-series, leaving the investigation of "nongeometric" and nonabelian orbifolds for the future.

Let γ be a generator of $\mathbb{Z}_n$. Its acts on the WZW field $g(z,\bar{z})$ via

$$g(z,\bar{z}) \to e^{-\frac{2\pi i}{n}\sigma_3} g(z,\bar{z})\,. \tag{4.1}$$

The right-moving WZW currents are invariant, while the left-moving currents transform as

$$J_L(\bar{z}) \to e^{-\frac{2\pi i}{n}\sigma_3} J_L(\bar{z}) e^{\frac{2\pi i}{n}\sigma_3}\,. \tag{4.2}$$

Thus the left-moving current algebra is broken down to $\mathrm{U}(1)$ for $n > 2$ and left intact for $n = 2$.

We must also choose the action of γ on the left-moving internal bosons $H_\alpha, \alpha = 1, \ldots, 16$. Without loss of generality we can take γ which acts by a shift,

$$H_\alpha \to H_\alpha + 2\pi v_\alpha\,. \tag{4.3}$$

v is defined modulo vectors from $\mathcal{L}$ where $\mathcal{L}$ is the lattice on which the H_α live. In addition, the vector v must satisfy $nv \in \mathcal{L}$. We will define $u = nv$; possible choices of u (and therefore v) are classified by $\mathcal{L}/n\mathcal{L}$. Geometrically, picking u amounts to picking a monodromy of the gauge bundle on $\mathbb{R}^4/\mathbb{Z}_n\backslash\{0\}$.

It is convenient to think about the monodromy as due to a point-like instanton sitting at the origin of $\mathbb{R}^4/\mathbb{Z}_n$. The instanton charge depends on the monodromy and can be determined by blowing up the orbifold. Then, as a consequence of the anomalous Bianchi identity $dH = \mathrm{Tr}\, R \wedge R - \mathrm{Tr}\, F \wedge F$, the orbifold plane has a fivebrane charge which depends on u.

From the worldsheet point of view, it is more convenient to fix the WZW level k (the analog of the fivebrane charge) and ask what choices of u are allowed by modular invariance. Below we show that modular invariance requires

$$k + \frac{u^2}{2} = 0 \pmod{n}\,. \tag{4.4}$$

Note that since $\mathcal{L}$ is an even lattice, u^2 is always even, and therefore there is no conflict between modular invariance and integrality of k.

In order to implement the $\mathbb{Z}_n$ action on the the WZW model, it is convenient to rewrite the left moving sector as a product of a parafermionic $\mathrm{SU}(2)/\mathrm{U}(1)$ coset and a free scalar field ξ. The orbifold group acts only on the free scalar. For details see e.g. [31]. The left-moving boson ξ is related to the current via

$$J_L^3 = \frac{ik}{2}\partial\xi\,.$$

[6] This remark applies equally well to type-II LSTs with (1,0) SUSY.

ξ has a unit radius and obeys an OPE of the form

$$\xi(\bar{z})\xi(0) = -\frac{2}{k}\log\bar{z} + \cdots .$$

γ acts on ξ via

$$\xi \to \xi + \frac{4\pi}{n}.$$

The partition function of the orbifolded theory can be written schematically as

$$Z_{orb} = \frac{1}{n}\sum_{p,q=0}^{n-1}(\gamma^p, \gamma^q)\,, \tag{4.5}$$

where (γ^p, γ^q) corresponds to the contribution twisted by γ^p with γ^q inserted in the trace. The untwisted partition function projected onto the invariant states is given by the terms with $p = 0$. The twist operator which takes us to the sector twisted by γ^w is given by

$$\Omega_w \sim e^{\frac{iw}{n}(k\xi + u\cdot H)}\,. \tag{4.6}$$

A primary from the $w^{\rm th}$ twisted sector has a generic form

$$\Omega_w(\bar{z})V_{j_R,m_R}(z)V_{j_L,m_L}(\bar{z})e^{i\alpha\cdot H}\,, \tag{4.7}$$

where $\alpha \in \mathcal{L}$. (For $\alpha = 0$ the last factor must be replaced with ∂H.) Requiring $h_L - h_R \in \mathbb{Z}$ we get

$$w\left(2m_L + \frac{wk}{n}\right) + w\alpha\cdot u + \frac{w^2u^2}{2n} = 0 \pmod{n}\,. \tag{4.8}$$

Setting $w = 1$ and taking into account that $2m_L \in \mathbb{Z}$ we obtain eq. (4.4). Let us define an integer ℓ by $\ell = (k + u^2/2)/n$. Requiring that operators in different sectors be mutually local we obtain a quantization condition on m_L in the $w^{\rm th}$ twisted sector:

$$2m_L + \alpha\cdot u + w\ell = 0 \pmod{n}\,. \tag{4.9}$$

With this quantization condition eq. (4.8) is satisfied for all w.

We now consider some special choices for u. The simplest choice is $u = 0$. Then the action of γ on H_α is trivial and the worldsheet CFT has unbroken G current algebra. A closely related CFT was considered in ref. [31]. From the space-time point of view $u = 0$ means that the monodromy of the flat connection on $\mathbb{R}^4/\mathbb{Z}_n\backslash\{0\}$ is taken to be trivial. For $n > 2$ we thus find an LST with global symmetry $\mathrm{U}(1)_L \times \mathrm{SU}(2)_R \times G$. For $n = 2$ γ acts trivially on the $\mathrm{SU}(2)_L$ currents and the symmetry is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$. The $n = 2$ model is identical to the D-series LSTs of section 2.

For $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ we may also take $u = (2, 0, \ldots, 0)$. For $n > 2$ the monodromy corresponding to this u breaks the left-moving internal current algebra from so(32) down to so(30) $\oplus$ u(1). The case $n = 2$ is special in that none of the gauge bosons of G are projected out and the gauge symmetry algebra is still so(32) and the $\mathrm{su}(2)_L$ also remains unbroken. Thus the symmetry algebra is the same as for the D-series LSTs. Moreover, for $n = 2$ the condition eq. (4.4) becomes $k + 2 = 0$ (mod 2), which is the same as for the D-series LSTs. Nonetheless, this theory is not identical to the D-series LST, because γ acts nontrivially on spinors of $\mathrm{Spin}(32)/\mathbb{Z}_2$. As a result, the global symmetry group of this theory is $(\mathrm{Spin}(32)/\mathbb{Z}_2 \times \mathrm{SU}(2)_L)/\mathbb{Z}_2$, where the second $\mathbb{Z}_2$ reverses the sign of the spinor of $\mathrm{Spin}(32)/\mathbb{Z}_2$ and generates the center of $\mathrm{SU}(2)_L$.

For $G = E_8 \times E_8$ an analogous choice is $u = (2, 0, \ldots, 0)$ for the first E_8 and $u = (0, \ldots, 0)$ for the second E_8. For $n > 2$ this leads to LSTs with global symmetry algebra $\mathrm{u}(1)_L \oplus \mathrm{u}(1) \oplus \mathrm{so}(14) \oplus e_8$. For $n = 2$ we get instead $\mathrm{su}(2)_L \oplus \mathrm{so}(16) \oplus e_8$.

None of the above models corresponds to fivebranes near a perturbative heterotic orbifold with the standard embedding of the spin connection into the gauge connection. The latter has a monodromy which breaks so(32) down to $\mathrm{su}(2) \oplus \mathrm{u}(1) \oplus \mathrm{so}(28)$ and $e_8 \oplus e_8$ down to $\mathrm{u}(1) \oplus e_7 \oplus e_8$. (For $n = 2$ it only breaks so(32) down to $\mathrm{su}(2) \oplus \mathrm{su}(2) \oplus \mathrm{so}(28)$ and $e_8 \oplus e_8$ down to $\mathrm{su}(2) \oplus e_7 \oplus e_8$.) To get such a spectrum of unbroken gauge bosons we need to set $u = (1, 1, 0, \ldots, 0)$ for $G = \mathrm{Spin}(32)/\mathbb{Z}_2$; for $G = E_8 \times E_8$ we need to set $u = (1, 1, 0, \ldots, 0)$ for the first E_8 and $u = (0, \ldots, 0)$ for the second E_8.

5. Conclusions

In this paper we constructed a variety of heterotic LSTs arising from heterotic string theory in linear dilaton backgrounds. For each affine Dynkin diagram of type A, D, or E we constructed a worldsheet CFT which is holographic to a heterotic (1, 0) LST with global symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times G$, where $\mathrm{SU}(2)_R$ is the R-symmetry and $G = \mathrm{Spin}(32)/\mathbb{Z}_2$ or $E_8 \times E_8$. The A-series are heterotic fivebranes in flat space. The D-series correspond to fivebranes near an $\mathbb{R}^4/\mathbb{Z}_2$ orbifold with a trivial gauge connection. The E-series do not admit a geometric description either in heterotic string theory or M-theory. In particular, we found that a single heterotic fivebrane admits a well-defined holographic dual (for either choice of G). This is in contrast with a single type-II fivebrane, which apparently does not lead to a decoupled Poincaré-invariant theory and does not admit a holographic description [17].

We studied the spectrum of chiral operators in these theories using worldsheet methods. We found that in all models there are operators which are singlets of G and are traceless symmetric tensors of $\mathrm{SO}(4) \sim \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The orders of the tensors are in one-to-one correspondence with the orders of the Casimirs of the corresponding ADE Lie group. This part of the spectrum is very similar to that of type-II LSTs [14]. There are also operators which are charged under G. However, comparison with the large N limit revealed that many chiral operators are missed by the worldsheet analysis. In particular, we did not find any self-dual three-forms expected for $E_8 \times E_8$ LSTs, or any evidence for the $\mathrm{Sp}(k+1)$ gauge group for $\mathrm{Spin}(32)/\mathbb{Z}_2$ LSTs.

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We compared the chiral operators of the LST describing $\mathrm{Spin}(32)/\mathbb{Z}_2$ heterotic fivebranes with chiral operators of the Yang-Mills theory describing type-I D5 branes. We found that the adjoints of $\mathrm{Spin}(32)/\mathbb{Z}_2$ match precisely between the two theories. The above-mentioned traceless symmetric tensors of $SO(4)$ also almost match, but for $N_5 > 1$ the LST has one more of them than the Yang-Mills theory. We suggested several possible viewpoints on this discrepancy. The most intriguing one is that along some flat directions of the LST certain modes become free in the Yang-Mills theory but not in the full LST. If this possibility is realized, then the linear dilaton backgrounds we considered describe heterotic LSTs infinitely far along such flat directions rather than at the origin of the moduli space.

Finally, we have constructed linear dilaton backgrounds which describe heterotic fivebranes near orbifold singularities of the form $\mathbb{R}^4/\mathbb{Z}_n$. The orbifold does not have to be a perturbative orbifold with zero fivebrane charge; rather, when the fivebrane charge of the orbifold plane is nonzero, modular invariance requires the level of the WZW model to be shifted appropriately (see eq. (4.4)). We also pointed out that it may be possible to construct nongeometric backgrounds with linear dilaton by gauging accidental symmetries of WZW models at special values of the level. It would be interesting to check whether modular-invariant orbifolds of this sort really exist.

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References

[1] E. Witten, *Some comments on string dynamics*, `hep-th/9507121`; *Five-branes and M-theory on an orbifold*, *Nucl. Phys.* **B 463** (1996) 383 [`hep-th/9512219`].

[2] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [`hep-th/9512059`].

[3] O.J. Ganor and A. Hanany, *Small E_8 instantons and tensionless noncritical strings*, *Nucl. Phys.* **B 474** (1996) 122 [`hep-th/9602120`].

[4] N. Seiberg and E. Witten, *Comments on string dynamics in six-dimensions*, *Nucl. Phys.* **B 471** (1996) 121 [`hep-th/9603003`].

[5] M.J. Duff, H. Lu and C. N. Pope, *Heterotic phase transitions and singularities of the gauge dyonic string*, *Phys. Lett.* **B 378** (1996) 101 [`hep-th/9603037`].

[6] R. Dijkgraaf, E. Verlinde and H. Verlinde, *BPS spectrum of the five-brane and black hole entropy*, *Nucl. Phys.* **B 486** (1997) 77 [hep-th/9603126]; *BPS quantization of the five-brane*, *Nucl. Phys.* **B 486** (1997) 89 [hep-th/9604055].

[7] M. Berkooz, M. Rozali and N. Seiberg, *Matrix description of M-theory on T^4 and T^5*, *Phys. Lett.* **B 408** (1997) 105 [hep-th/9704089].

[8] N. Seiberg, *New theories in six-dimensions and matrix description of m theory on T^5 and T^5/Z_2*, *Phys. Lett.* **B 408** (1997) 98 [hep-th/9705221].

[9] K. Intriligator, *New string theories in six-dimensions via branes at orbifold singularities*, *Adv. Theor. Math. Phys.* **1** (1998) 271 [hep-th/9708117].

[10] E. Witten, *New "gauge" theories in six-dimensions*, *J. High Energy Phys.* **01** (1998) 001 [hep-th/9710065].

[11] O. Aharony, M. Berkooz, S. Kachru, N. Seiberg and E. Silverstein, *Matrix description of interacting theories in six dimensions*, *Adv. Theor. Math. Phys.* **1** (1998) 148 [hep-th/9707079].

[12] E. Witten, *On the conformal field theory of the Higgs branch*, *J. High Energy Phys.* **07** (1997) 003, [hep-th/9707093].

[13] S. Sethi, *The matrix formulation of type-IIB five-branes*, *Nucl. Phys.* **B 523** (1998) 158 [hep-th/9710005].

[14] O. Aharony, M. Berkooz, D. Kutasov and N. Seiberg, *Linear dilatons, NS5-branes and holography*, *J. High Energy Phys.* **10** (1998) 004 [hep-th/9808149].

[15] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[16] S. Minwalla and N. Seiberg, *Comments on the IIA NS5-brane*, *J. High Energy Phys.* **06** (1999) 007 [hep-th/9904142].

[17] D.-E. Diaconescu and N. Seiberg, *The coulomb branch of $(4,4)$ supersymmetric field theories in two dimensions*, *J. High Energy Phys.* **07** (1997) 001 [hep-th/9707158].

[18] A. Cappelli, C. Itzykson and J.B. Zuber, *Modular invariant partition functions in two-dimensions*, *Nucl. Phys.* **B 280** (1987) 445; *The ADE classification of minimal and $A_1(1)$ conformal invariant theories, Comm. Math. Phys.* **113** (1987) 1.

[19] C.V. Johnson, *On the $(0,4)$ conformal field theory of the throat*, *Mod. Phys. Lett.* **A 13** (1998) 2463 [hep-th/9804201].

[20] D.-E. Diaconescu and J. Gomis, *Neveu-Schwarz 5-branes at orbifold singularities and holography*, *Nucl. Phys.* **B 548** (1999) 258 [hep-th/9810132].

[21] J. Curtis G. Callan, J.A. Harvey and A. Strominger, *World sheet approach to heterotic instantons and solitons*, *Nucl. Phys.* **B 359** (1991) 611.

[22] J. Curtis G. Callan, J.A. Harvey and A. Strominger, *Worldbrane actions for string solitons*, *Nucl. Phys.* **B 367** (1991) 60.

[23] D. Friedan, E. Martinec and S. Shenker, *Conformal invariance, supersymmetry and string theory*, *Nucl. Phys.* **B 271** (1986) 93.

[24] M. Berkooz, *A supergravity dual of a* $(1,0)$ *field theory in six- dimensions*, *Phys. Lett.* **B 437** (1998) 315 [hep-th/9802195].

[25] E.G. Gimon and C. Popescu, *The operator spectrum of the six-dimensional* $(1,0)$ *theory*, *J. High Energy Phys.* **04** (1999) 018 [hep-th/9901048].

[26] D. Gepner, *Space-time supersymmetry in compactified string theory and superconformal models*, *Nucl. Phys.* **B 296** (1988) 757.

[27] P.S. Aspinwall, *Pointlike instantons and the* $\mathrm{Spin}(32)/\mathbb{Z}_2$ *heterotic string*, *Nucl. Phys.* **B 496** (1997) 149 [hep-th/9612108].

[28] K. Intriligator, *Rg fixed points in six-dimensions via branes at orbifold singularities*, *Nucl. Phys.* **B 496** (1997) 177 [hep-th/9702038].

[29] P. Horava and E. Witten, *Heterotic and type-I string dynamics from eleven- dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209].

[30] P. Bouwknegt and W. Nahm, *Realizations of the exceptional modular invariant A-(1) partition functions*, *Phys. Lett.* **B 184** (1987) 359.

[31] D. Kutasov, F. Larsen and R.G. Leigh, *String theory in magnetic monopole backgrounds*, *Nucl. Phys.* **B 550** (1999) 183 [hep-th/9812027].

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